

Exercise 79

Find h' in terms of f' and g' .

$$h(x) = \frac{f(x)g(x)}{f(x) + g(x)}$$

Solution

Calculate $h'(x)$ by using the product and quotient rules.

$$\begin{aligned} h'(x) &= \frac{d}{dx} \left[\frac{f(x)g(x)}{f(x) + g(x)} \right] \\ &= \frac{\left[\frac{d}{dx}[f(x)g(x)] \right] [f(x) + g(x)] - \left[\frac{d}{dx}[f(x) + g(x)] \right] [f(x)g(x)]}{[f(x) + g(x)]^2} \\ &= \frac{[f'(x)g(x) + f(x)g'(x)][f(x) + g(x)] - [f'(x) + g'(x)][f(x)g(x)]}{[f(x) + g(x)]^2} \\ &= \frac{\cancel{f'(x)f(x)g(x)} + f'(x)[g(x)]^2 + [f(x)]^2g'(x) + \cancel{f(x)g'(x)g(x)} - \cancel{f'(x)f(x)g(x)} - \cancel{f(x)g'(x)g(x)}}{[f(x) + g(x)]^2} \\ &= \frac{f'(x)[g(x)]^2 + [f(x)]^2g'(x)}{[f(x) + g(x)]^2} \end{aligned}$$